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Fig. 1 Failure of Teton Dam, on the Teton river, in eastern Idaho, on June 5, 1976, possibly a rare instance of a mixed kinematic-dynamic wave.

MIXED KINEMATIC-DYNAMIC WAVES DEBUNKED

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ABSTRACT. We answer the question as to whether the mixed kinematic-dynamic wave is generally too strongly diffusive to be considered a practical flood wave. We show that in the great majority of cases, these mixed waves are not there for us to calculate them! Their typical midsize obliges them to attenuate very quickly, with their mass joining the underlying kinematic or diffusion wave, which continues to grow in both size and permanence as it propagates downstream. Since a diffusion wave will calculate diffusion, including the case of zero diffusion, it is clear that the solution of a diffusion wave encompasses that of a kinematic wave. Therefore, the diffusion wave is postulated as the flood wave *par excellence*, i.e., the wave type generally indicated for use in practical flood routing applications.

1. INTRODUCTION

When modeling flood waves, often the first question that comes to mind is: What type of wave should I use? Kinematic and diffusion waves are well established. Moreover, it is recognized that the dynamic waves of Lagrange do not lend themselves to flood routing applications. The question remains: How good are the *mixed kinematic-dynamic waves* notably elaborated by **Fread**? Note that we are here referring to the solution of the complete St. Venant equations of unsteady open-channel flow in one spatial dimension.

Over the past 50 years, the approach that seems to have prevailed in some quarters is the following: "Forget about the various types of waves; let's use the complete solution of the St. Venant equations in **all** applications, and let the computer do the number crunching!" We note here that theory and experience have confirmed that this approach is generally ill-fated. Hydrodynamic theory indicates otherwise; moreover, practical applications confirm the shortsightedness of placing all eggs in one basket. In this article, we strive to debunk the notion that the mixed kinematic-dynamic wave should be the **only way** to model flood wave propagation.

We aim to show that the sole use of the mixed wave approach is at best futile, and at worst, wrong; and very likely to lead to wasted time and resources. For added clarity, in the following section we list the various types of waves in current use, while elaborating on their nature and properties.

2. TYPES OF WAVES

In one-dimensional unsteady free-surface flow, the following four wave types are in general use: (1) kinematic waves; (2) diffusion waves; (3) mixed waves; and (4) dynamic waves. Kinematic waves exclude the inertia and pressure-gradient terms; diffusion waves exclude only the inertia terms; mixed kinematic-diffusion waves (the complete solution) exclude no terms, and dynamic waves exclude the friction and gravity terms (Table 1). The excluded terms are removed from consideration because they are too small to materially affect the properties of the wave in question.

Table 1. Types of waves in one-dimensional unsteady free-surface flow.

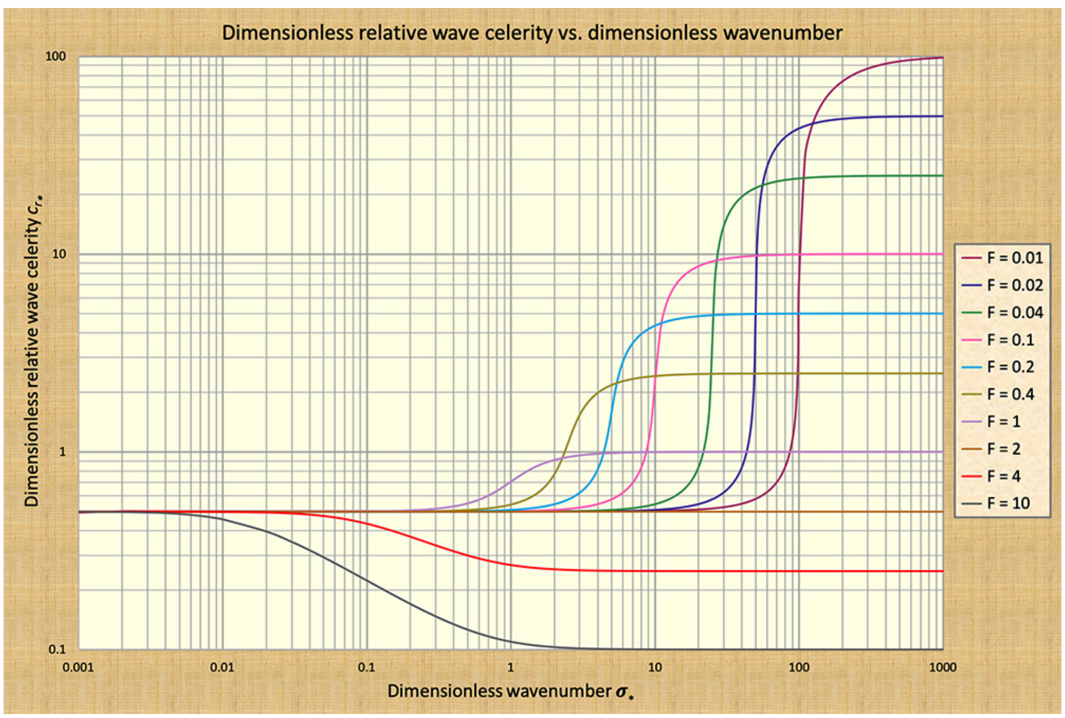
No.	Wave type	Terms of the equation of motion participating in the wave description					Common name
		Local inertia	Convective inertia	Pressure gradient	Friction	Gravity	
1	Kinematic without diffusion				✓	✓	Kinematic
2	Kinematic with diffusion			✓	✓	✓	Diffusion
3	Mixed kinematic-dynamic	✓	✓	✓	✓	✓	Mixed
4	Dynamic	✓	✓	✓			Dynamic

3. WAVE PROPERTIES

The properties of these wave types have been examined in detail by **Ponce and Simons (1977)** (Fig. 2). These authors used linear stability theory to determine celerity and attenuation functions for: (1) kinematic, (2) diffusion, (3) mixed kinematic-dynamic, and (4) dynamic. The unifying element is seen to be the dimensionless wavenumber $\boldsymbol{\sigma_*}$, defined by multiplying the applicable wavenumber ($2\pi / L$) times the reference channel length L_o, i.e., the length of channel that it would take the equilibrium flow to drop a head equal to its depth.

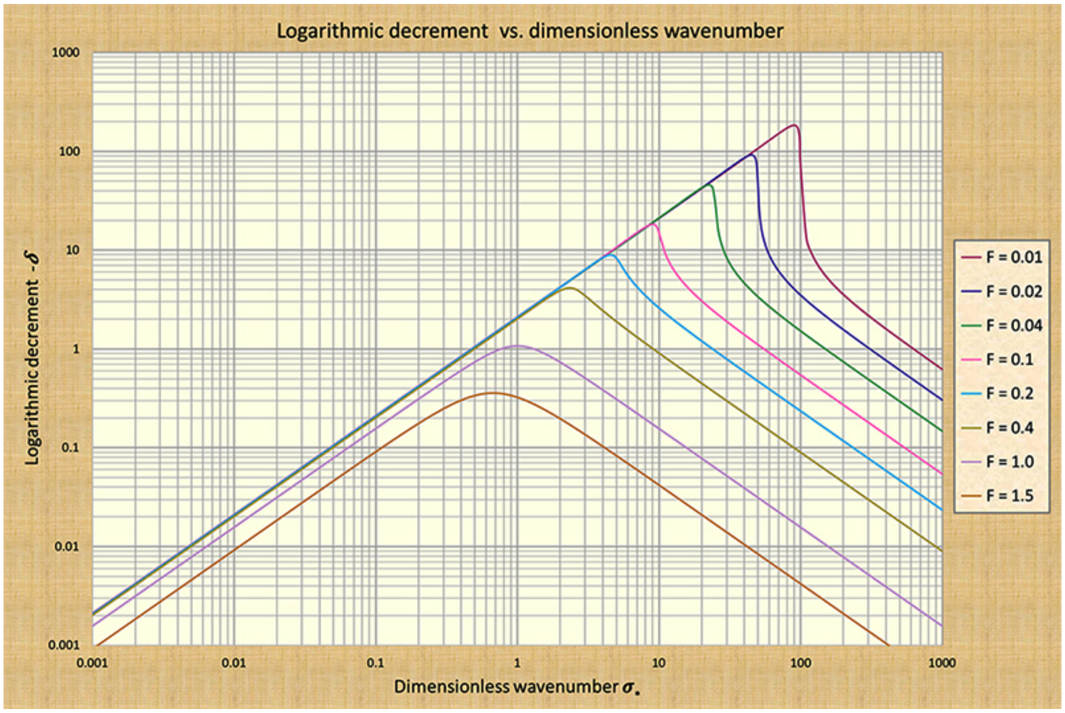
Kinematic waves are those of **Seddon (1900)**, while dynamic waves are those of Lagrange (1788). Mixed kinematic-dynamic waves are those lying along the middle-to-right of the wavenumber spectrum (Fig. 2). These waves, hereafter referred to as mixed waves, were featured in the numerical models developed beginning in the 1970s to solve the complete St. Venant equations; see, for instance, Fread (1985). These models have been widely referred to as "dynamic wave" models, although the misnomer has led to some confusion with the long-established Lagrange (1788) waves.

Diffusion waves lie to the right of kinematic waves and to the left of mixed kinematic-dynamic waves in the wavenumber spectrum (Fig. 2). Unlike kinematic waves, which feature zero diffusion, diffusion waves have a small but perceptible amount of diffusion. However, this diffusion is small compared to that of the mixed waves (Fig. 3). We note that the inclusion of the pressure-gradient term (Table 1) is directly responsible for the diffusion.



Ponce and Simons (1977)

Fig. 2 Dimensionless relative wave celerity c_{r*} vs dimensionless wavenumber σ_*.



Ponce and Simons (1977)

Fig. 3 Logarithmic decrement $-\delta$ vs dimensionless wavenumber σ_*.

Figure 2 shows that Seddon's kinematic waves, lying toward the left of the dimensionless wavenumber spectrum, feature a constant wave celerity and are, therefore, nondiffusive. Following the same rationale, Lagrange's dynamic waves, lying toward the right, are also nondiffusive. However, the mixed waves, lying toward the middle-to-right and featuring sharply varying celerity, are shown to be strongly diffusive. The amount of diffusion, characterized by the logarithmic decrement δ, varies with the prevailing Froude number (Fig. 3) (Wylie, 1966) (see also **Box A**). Greater diffusion corresponds to the lower Froude numbers, provided the latter remains below the threshold value $\mathbf{F}$ = 2, applicable for Chezy friction in hydraulically wide channels (Figs. 2 and 3).

4. KINEMATIC WAVES

A kinematic wave may be indeed regarded as the quintessential flood wave. Theory tells us that a kinematic wave does not attenuate. Practical experience would indicate that if a wave attenuates very quickly, it is most likely **not** a flood wave. Mathematically, we reckon that the constancy of the wave celerity, across a specified range of small dimensionless wavenumbers ($0.001 \leq \sigma_* \leq 0.01$), is a sure indication of the presence of a kinematic wave (**Ponce and Simons, 1977**). Kinematic waves diffuse either imperceptibly or not at all. However, they may suffer change of shape due to nonlinearities, the latter being a process by which different discharges travel with different celerities (**Ponce and Windingland, 1985**).

At this juncture, we endeavor to quote **Lighthill and Whitham (1955)**, who established the foundation of kinematic wave theory. They keenly observed: "In some applications, including the case of flood waves, kinematic waves and dynamic waves are both possible together. However, the dynamic waves have a much higher wave velocity and also a rapid attenuation. Hence, although any disturbance sends some signal downstream at the ordinary wave velocity for long gravity waves [*sic*], this signal is too weak to be noticed at any considerable distance downstream, and the main signal arrives in the form of a kinematic wave at a much slower velocity."

5. DIFFUSION WAVES

Unlike kinematic waves, diffusion waves are subject to a small amount of diffusion. They lie *immediately to the right* of kinematic waves in the dimensionless wavenumber spectrum, properly within the range $0.01 \leq \sigma_* \leq 0.17$ (Fig. 2) (**Ponce, 2024**). The value $\sigma_* = 0.01$ depicts 2.1% wave diffusion, admittedly a relatively small amount, while the value $\sigma_* = 0.17$ depicts 30% wave diffusion. The latter is regarded as a threshold between diffusion and mixed waves (Natural Environment Research Council, 1975).

Typical flood waves diffuse somewhat; therefore, diffusion waves are indeed a practical model of flood wave propagation. They complement kinematic waves rather nicely, while finding their best application in cases where wave diffusion is appreciable and its calculation is deemed necessary. There is, however, a catch. While not originally intended, conventional kinematic wave models may actually show **some** wave diffusion. This diffusion is artificial, and in no way related to the diffusion that would accrue if the wave were to be an actual diffusion wave. Therefore, the procedure is hit and miss as far as the true wave diffusion is concerned. The artificial diffusion in question, indeed "numerical diffusion", arises from the discrete nature of the grid and the associated lack of numerical convergence.

The matter of how to best handle the numerical diffusion has been resolved by **Cunge (1969)**, who proposed a match of the numerical diffusion of the scheme itself with the physical diffusion of the related kinematic wave equation with diffusion, i.e., the diffusion wave equation. This development led to the Muskingum-Cunge method of flood routing, a physically-based alternative to the well-known Muskingum method (**Ponce, 2014a**).

6. DYNAMIC WAVES

Classical dynamic waves are those of Lagrange (1788). More recently, Fread (1985) and others have referred to the mixed kinematic-dynamic waves as "dynamic" waves, while herein we refer to them simply as "mixed" waves. The semantic confusion is judged to be unfortunate. In an attempt to fix the problem, in this article we use the adjective "dynamic" to refer solely to the Lagrange waves.

Dynamic waves feature a constant wave celerity for dimensionless wavenumber $\sigma_* \geq 100$, for most Froude numbers, and $\sigma_* \geq 1000$ for all Froude numbers (Fig. 2). This means conclusively, as with kinematic waves, that the dynamic waves of Lagrange are *not* subject to diffusion.

The dynamic waves of Lagrange are not the typical flood waves. Their size is too small to constitute a veritable flood risk. Their appplication is restricted to short wave propagation in irrigation and power canals, where the scale of the disturbance is such that it may be actually seen, or perceived, with the naked eye. Unlike flood waves, which are mass waves that feature only **one wave** traveling downstream, classical dynamic waves are energy waves, which feature **two waves**, traveling in opposite directions under subcritical flow, and in only one direction (downstream) in supercritical flow.

For enhanced clarity, a parenthetical comment regarding the cause of wave diffusion is advisable here. Diffusion is produced by the interaction of the pressure gradient with the friction and gravity terms

(Table 1, Row 2). More precisely defined, diffusion is produced by the interaction of the *non-kinematic* (read "*dynamic*") terms (inertia and/or the pressure gradient), with the *kinematic* terms (friction and gravity) (Table 1, Row 3) (**Ponce, 1982**). The amount of wave diffusion is proportional to the interaction between kinematic and dynamic terms of the equation of motion. Lack of kinematic terms results in zero diffusion, depicted by the curve to the far right of Fig. 2; conversely, lack of dynamic terms also results in zero diffusion, depicted by the curve to the far left of Fig. 2.

7. MIXED WAVES

At this juncture, it remains for us to discuss the only other wave type left: The mixed kinematic-dynamic wave, for short, the "mixed" wave of unsteady open-channel flow. Since, by definition, this wave features **both** kinematic and dynamic components in comparable amounts, it follows that it must be strongly diffusive.

The answer to this question is Yes! The mixed kinematic-dynamic wave is indeed *very strongly diffusive*. In fact, it is the most diffusive of all the wave types considered in this article! Given this fact, the question that remains is if the mixed wave may be construed as a flood wave, or not. To answer this question precisely, we resort once again to the illuminating work of **Ponce and Simons (1977)** and to their analytical calculation of celerity and attenuation functions for all types of shallow-water waves. The amounts of wave attenuation calculated by Ponce and Simons (see detail in **Box A**) are depicted in Fig. 3 and complemented with Table 2.

Box A. The logarithmic decrement *δ*.

The logarithmic decrement δ is defined as the amount of wave attenuation (the reduction in wave amplitude A) experienced by a sinusoidal perturbation in the time elapsed from t = 0 to t = 1 period, i.e., within *one* period of propagation. In other words: $A_1 = A_0\ e^{\delta}$. It is a convenient, albeit expedient, way to analyze and compare wave attenuation amounts. To explain it mathematically, we endeavor to quote here directly from the original source **(Ponce and Simons, 1977, Page 1464)**:

- The wave attenuation follows an exponential law in which the amplitude at a given time t is equal to the initial amplitude at time t_o multiplied by $(e^{\beta_{*I}\, t_*})$, in which $t_* = (t - t_o)\ u_o / L_o$.
- When comparing wave amplitudes after one propagation period, $t_* = T\ u_o/L_o$, or likewise, $t_* = 2\ \pi\ /\ |\beta_{*R}|$. Thus, [added here for clarity] the logarithmic decrement δ is defined as $\delta = \beta_*\ T\ u_{\ o}\ /\ L_o$, or $\delta = 2\ \pi\ \beta_{*I}\ /\ |\beta_{*R}|$.
- The value of δ is a measure of the rate at which the unsteady component of the motion changes upon propagation. For δ positive, amplification (i.e., a logarithmic *increment*) sets in; for δ negative, the motion attenuates and dies away (i.e., a logarithmic *decrement*).

Within the σ_* range shown in Fig. 3, for subcritical flows (**F** < 1), where the attenuation is shown to be stronger (greater values of δ), the logarithmic decrement is seen to vary from a low of δ = 0.0021 for σ_* = 0.001 (Table 2, Line 1), to a peak of δ = 180 for σ_* = 90 (Table 2, Line 6).

Table 2, Line 0 (emphasized with yellow background) shows a very small amount of wave attenuation, 0.02%, or **0.0002**, associated with a very low value of logarithmic decrement δ = 0.00021 corresponding to σ_* = 0.0001, which ostensibly lies outside of the range shown in Fig. 3.

Table 2, Line 3a (with yellow background) purposely depicts a wave attenuation of **0.3** (Col. 4), that is, a 30% decay of the wave amplitude, a threshold value widely considered as the division between diffusion waves (less than or equal to 30% attenuation) and mixed waves (more than 30% attenuation) (Natural Environment Research Council, 1975). This threshold corresponds to a value of σ_* = **0.17**.

Table 2, Line 4a (with yellow background) purposely depicts a wave attenuation of **0.99** (Col. 4), that is, a 99% decay of the wave amplitude, an attenuation value which nearly erases the wave altogether! This attenuation amount corresponds to a value of σ_* = **2**.

Table 2, Line 5a (with yellow background) purposely depicts a wave attenuation of **1.0** (Col. 4), that is, a 100% decay of the wave amplitude, an attenuation value which erases the wave altogether! This attenuation amount corresponds to a value of σ_* = **90**.

Table 2, Col. 5 shows the indicated wave types, from kinematic, with very small attenuation (0.0002), to diffusion, with small to medium attenuation (0.0021 to 0.1894), to mixed wave, with large to very large attenuation (0.3 to 0.9999). Table 2, Lines 5 and 6 depict a nonexisting wave; the wave having disappeared completely, with its mass going on to form part of the underlying, or equilibrium, flow.

Table 2. Amounts of wave attenuation across the dimensionless wavenumber spectrum (σ_* ≤ 90).

[1]	[2]	[3]	[4]	[5]	[6]
No.	**Dimensionless wavenumber σ_***	**Logarithmic decrement δ**	**e^{δ}**	**Wave attenuation $A = (1 - e^{\delta})$**	**Wave type**
0	**0.0001**	0.00021	0.9998	**0.0002**	Kinematic
1	0.001	0.0021	0.9979	0.0021	Kinematic to diffusion
2	0.01	0.021	0.9792	0.0208	Diffusion
3	0.1	0.21	0.8106	0.1894	Diffusion
3a	**0.17**	0.357	0.7	**0.3**	Diffusion to mixed
4	1.	2.1	0.1224	0.8776	Mixed
4a	**2.**	4.6	0.01	**0.99**	Mixed
5	10.	21.	0.	1.0	No wave

5a	**90.**	180.	0.	**1.0**	No wave

Given that wave attenuation is $A = (1 - e^{\delta})$ (Table 2, Col. 5), the results of Table 2 lead to the following waves and corresponding ranges:

- **Kinematic waves:** $\sigma_* \le 0.001$; $A \le 0.0021$
- **Diffusion waves:** $0.001 < \sigma_* \le 0.17$; $0.0021 \le A \le 0.3$
- **Mixed waves:** $0.17 < \sigma_* \le 2$; $0.3 \le A \le 0.99$
- **No wave:** $\sigma_* > 2$; $A = 1$.

We conclude that most, if not all, mixed waves would have effectively lost all their strength in most cases of practical interest. They lose their strength rapidly due to their highly diffusive nature, the latter due to the competition between kinematic and dynamic terms (read forces) that are comparable in size. It follows that mixed waves lack a basic property of a flood wave, namely, its *permanency*, which is characterized by its mild or very mild amount of attenuation (diffusion). Thus, we argue that, in general, mixed waves may not be construed as flood waves.

8. DAM-BREACH FLOOD WAVES

Every rule is likely to call for an exception. In the previous section (Section 7), we presented an elaborate mathematical rationale for why the mixed wave is not likely to apply for the case of a general flood, i.e., one that is subject to very little or no attenuation. Yet we reckon that there is one particular flood wave that actually may diffuse appreciably. This is the case of a dam-breach flood wave.

Typically, the flood wave produced by the breaching of an earthen embankment is sudden, lasting about 3 hr, a sure candidate for strong wave diffusion. A case in point: Of 24 dam failures in the United States documented by **Taher-Shamsi *et al.* (2003)**, 17 of them failed in 3 hr or less.

Such flood waves (Fig. 4) are apt to fall under the category of mixed wave or, at the very least, be a *strongly diffusive* diffusion wave, with attenuation $A \cong 0.3$ (see Table 2, Line 3a). Fortunately for all of us, instances of dam breaches are rare and far between.

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Fig. 4 Failure of Teton Dam, on the Teton river, in eastern Idaho, on June 5, 1976, possibly a rare instance of a mixed kinematic-dynamic wave.

9. MODELING FLOOD WAVES

This section elaborates on ways to model flood waves. It seeks to answer the question: Now that I chose a type of wave, how should I proceed? What actual tool should I use in a real practical situation? This section is divided into three parts: (1) kinematic waves, (2) diffusion waves, and (3) mixed kinematic-dynamic waves. The classical dynamic waves of Lagrange described in Section 6 lie outside of the scope of this section.

Kinematic waves

Kinematic wave modeling may be performed in two ways. The first way is to realize that a true kinematic wave does not attenuate; therefore, subsidence, or diffusion, is out of the question. Still, the wave is actually moving downstream with a certain celerity, and that speed is subject to calculation. Indeed, that speed is Seddon's celerity, which states that the velocity of a flood wave at a given cross-section is equal to the slope of the rating curve (dQ/dy) divided by the stream or channel top width (T) (**Ponce, 2014b: Eq. 10-60**). Yet an alternate way of expressing Seddon's celerity is: $c = \beta u$, in which u = mean flow velocity, and β is the exponent of the discharge-flow area rating ($Q = \alpha A^{\beta}$) (**Ponce, 2014b: Eqs. 10-52 and 10-58**).

The simplicity of Seddon's celerity is remarkable, providing a ready tool to assess flood movement with a minimum of computational effort. **Box B** describes an example of the application of the concept.

Box B. Length of the flood wave in the Upper Paraguay river at Porto Murtinho, Matto Grosso, Brazil.

The calculation presented here allows a bit of reflection on the possible maximum size of a flood wave. The calculation is not intended to be accurate; it is given here only for general reference on the nature of flood waves in large tropical rivers.

The objective is to calculate the length of the flood wave on the Upper Paraguay river at Porto Murtinho, Mato Grosso de Sul, Brazil. This unique site features the longest possible flood wave period, estimated at one (1) year. The valley is that of the Pantanal of Mato Grosso (*Great Swampland of Mato Grosso*), which is wholly contained within Central Western Brazil and neighboring Eastern Bolivia. The distance along the river is 1,266 km, measured from the upstream location at Caceres, Mato Grosso, to the downstream location at Porto Murtinho, Mato Grosso de Sul (**Ponce, 1995**).

- Flood wave period: $T = 365$ d × (86,400 s/d) = 31,536,000 s
- Average stream velocity, based on field experience: $u = 0.1$ m/s
- Exponent β of the discharge-flow area rating (estimated) ($Q = \alpha A^{\beta}$): $\beta = 5/3$
- Flood wave celerity: $c = \beta\, u = (5/3) \times 0.1 = 0.167$ m/s
- Flood wavelength: $L = cT = 0.167$ m/s × (31,536,000 s) = 5,266,512 m
- **Flood wavelength at Porto Murtinho: L = 5,266 km.**

Fig. 5 Upper Paraguay river at Porto Murtinho, Mato Grosso do Sul, Brazil, featuring a flood hydrograph typically lasting one year.

The second way to perform kinematic wave modeling is to use a numerical model, many of which exist in various forms, in the literature and elsewhere. These models, however, suffer from a decided conundrum: How to properly model a kinematic wave without introducing a certain amount of numerical diffusion associated itself with the finite grid size . [Note that a kinematic wave proper is not supposed to have *any* diffusion! See Section 4]. The diffusion in question is *uncontrolled;* its existence may be confirmed by running the model for two different grid resolutions. This exercise will invariable result in *two different answers*, begging the question of which is the correct one (**Ponce, 1986**).

There does not appear to be a way out of this difficulty. At this juncture, the best that can be stated is that, for a sufficiently fine grid resolution, the numerical diffusion should reduce itself to where it may not be of much concern in a given practical application.

Diffusion waves

Unlike kinematic waves, which are governed by a first-order differential equation, describing only convection, diffusion waves are governed by a second-order equation, describing convection *and* diffusion. **Hayami (1951)** pioneered the development of diffusion wave theory by combining the equations of water continuity and motion, excluding the inertia terms (Table 1, Line 2), into a second-order convection-diffusion equation. This methodology has been widely referred to in the literature as Hayami's *diffusion analogy* (**Ponce, 2014a**). In flood wave modeling, the numerical solution of Hayami's second-order convection-diffusion equation provides a solution of a diffusion wave.

Cunge (1969) developed a convenient and practical alternative to Hayami's approach by solving the first-order kinematic wave equation numerically, while at the same time relating the amount of numerical diffusion produced by the finite grid size, to the actual physical diffusion of the second-order convection-diffusion equation of Hayami. Cunge observed that his flood routing methodology resembled the classical Muskingum method of McCarthy (1938), as cited by Chow (1959). More importantly, however, Cunge was able to tie in the numerical diffusion of the scheme itself to the physical diffusion of the flood wave in question, thus, rendering the procedure essentially grid-independent. The latter has been widely referred to as Muskingum-Cunge method (Natural Environment Research Council, 1975; **Ponce and Yevjevich, 1978)**.

The avowed feature of *grid independence* does wonders to set apart the Muskingum-Cunge method from existing kinematic wave numerical solutions, which ostensibly suffer from grid dependence. Since the solution of the diffusion wave equation contains, i.e., it encompasses the solution of the kinematic wave equation, the Cunge procedure may actually replace both Hayami's second-order solution and the conventional grid-dependent kinematic wave's first-order numerical solution. Thus, the Muskingum-Cunge method may be regarded as the method of choice to model flood waves numerically, with a reasonable expectation of accuracy, since the solution is, for all practical purposes, *almost* of second order (**Cunge, 1969**; **Ponce and Yevjevich, 1978**).

Mixed kinematic-dynamic waves

Mixed waves comprise all terms in the governing equations of water continuity and motion, that is, the St. Venant equations (Table, 1, Line 3). The inclusion of the inertia terms is indeed forceful, but is not without its pitfalls. The resulting wave is *dynamic*, characteristically of second order, therefore featuring *two* component waves, which travel in different directions, one upstream and the other downstream in subcritical flow, and in the same direction (downstream) in supercritical flow. The existence of two solutions throws a monkey wrench in the avowed purpose of flood routing, which ostensibly is to calculate the propagation of the primary wave, i.e., specifically the **one** that travels *downstream*.

Another significant pitfall is that the numerical solution of the complete St. Venant equations represents an order-of-magnitude increase in complexity in the formulation and actual performance of the numerical analog chosen to model the full equations. A scheme that appears to be widely favored by practitioners is the Preissmann box scheme (**Ponce *et al.*, 1978**). Theoretically, this scheme should provide second-order accuracy, if only its elements (namely, the temporal and spatial derivatives) are perfectly centered within the box, with a weighting factor $\theta = 0.5$. In practice, however, center-weighing the Preissmann scheme does not work, because it leads to strong numerical instabilities, which eventually render it inoperable. An expedient way out of this predicament has been to use $\theta > 0.5$, typically in the range 0.55-0.60, to stabilize the scheme by providing a certain amount of numerical diffusion to control the computation. Greater values of θ, in the range 0.6-1.0, provide increasing amounts of numerical diffusion, but this is *always* at the expense of increased nonconvergence (Fig. 6). Thus, the methodology is seen to degrade to first-order, compromising the original advantage predicated on the use of a complete "dynamic wave" model (i.e., our mixed wave model).

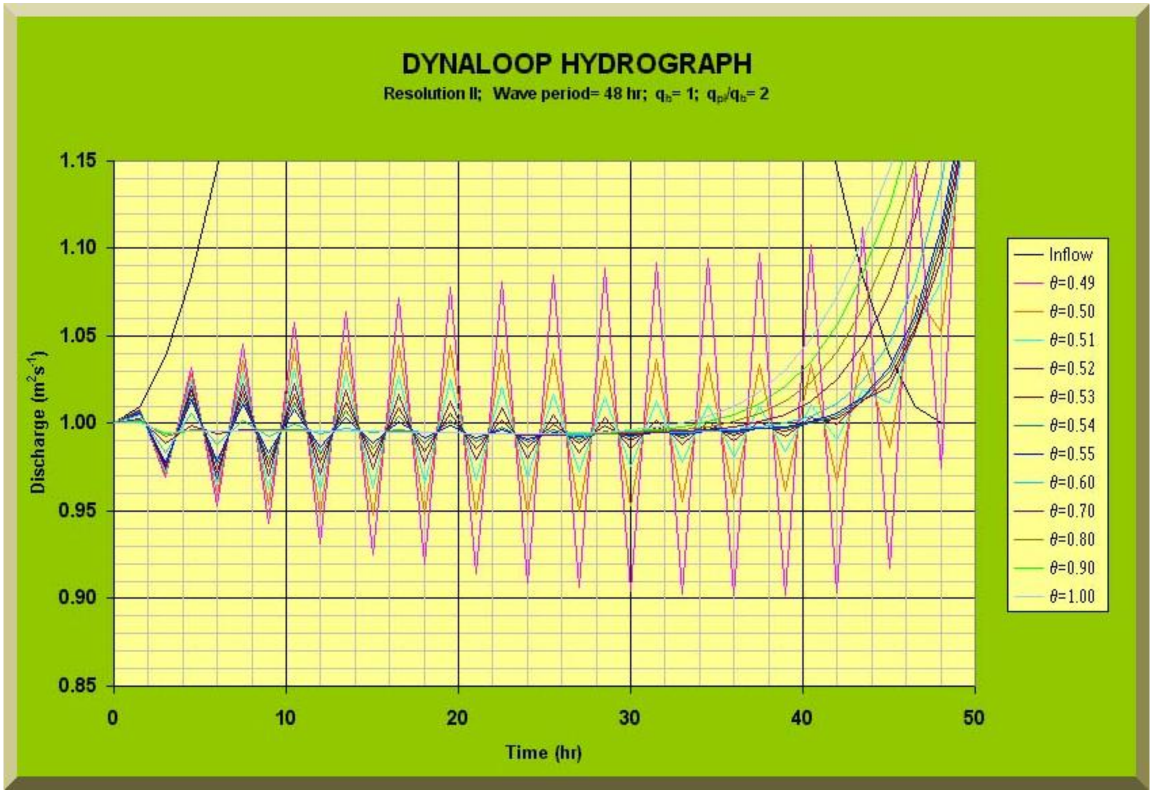

Fig. 6 Numerical instability generated by the use of the Preissmann scheme with a sinusoidal flood wave, with the weighting factor varying in the range 0.49-1.0. [Hydrograph input is partially shown in black color, while output at the leading edge of the hydrograph is shown in colors varying with *θ*]. Note that while oscillations at the base decrease with an increase in *θ*, thus enhancing stability, this increase leads to a faster output hydrograph peak (the trend shown only in the output hydrograph rise), showing increasing nonconvergence.

Yet another significant pitfall of the numerical solution of the St. Venant equations is that the model *does require* a downstream boundary condition to proceed. This fact was identified early by **Abbott (1976): Extract, Page 276** as a decided limitation, although later proponents of the methodology have apparently failed to pay due attention to this shortcoming. By nature, the method generates looped rating curves at internal computational points, forcing the need to also specify a looped rating at the downstream boundary. Clearly, the latter requirement is tantamount to "knowing the solution beforehand." An expedient way out of this difficulty has been to specify a *single rating* at the downstream boundary and to hope for the best! However, this procedure, while convenient because it helps solve the riddle, constitutes one more proof of why the methodology does not live up to its expectations. The feeling of "Is the wave dynamic or not?" remains to haunt those that continue to show confidence in the procedure.

We point out that the comments of this subsection purposely exclude U.S. government software such as the *U.S. Army Corps of Engineers River Analysis System*, widely known as HEC-RAS (**Wikipedia: HEC-RAS**). This comprehensive hydraulic software features, as one of its several components, a numerical model of the St. Venant equations using an implicit finite difference scheme. Tools such as HEC-RAS remain popular in practice because they are supported by the federal government, with other considerations being secondary in nature.

To sum up, by now it must be widely apparent that the mixed kinematic-dynamic wave is not what its users had originally in mind. The mixed wave is shown to be fraught with difficulties, the least of them being the realization of whether the said wave is there or not for us to calculate it! More commonly, the modeler will face other problems, of both a numerical and physical nature, which will have the net effect of casting doubts on the accuracy and practicality of the overall procedure.

10. ANALYSIS AND CONCLUSIONS

We have analyzed the celerity and attenuation properties of four types of shallow-water waves currently in use in hydraulic engineering: (1) kinematic, (2) diffusion, (3) mixed kinematic-dynamic, and (4) dynamic. Kinematic waves are massive (read, "large") and nondiffusive; diffusion waves are massive and diffusive. Mixed kinematic-diffusion waves, herein referred to simply as *mixed waves*, are relatively midsize (see Figs. 2 and 3) and may be shown to be strongly diffusive, while the dynamic waves of Lagrange are small and nondiffusive. The first two wave types, kinematic and diffusion, due to their large size and avowed permanence, may be construed as typical flood waves. The fourth type, the dynamic wave of Lagrange, is too small to be considered a flood wave.

We have sought to answer the question of whether the mixed wave is generally too strongly diffusive to be considered a practical flood wave. The answer is **Yes!** In the great majority of cases, the mixed waves may not be there for us to calculate them! Their typical midsize obliges them to attenuate very quickly, with their mass eventually joining the underlying kinematic or diffusion wave, which continues to grow in both size and permanence as it propagates downstream.

Note that only in the extremely unusual case of a dam-breach flood wave could we be actually confronted with the case of a mixed flood wave. A dam-breach flood wave is characteristically sudden, poised by Nature to be a mixed wave, an unusual type of flood wave [The experience of the Teton dam failure (Fig. 4) is a case in point]. Professionals in charge of forecasting or hindcasting a dam-breach flood wave would be keen to keep this in mind. For all other flood wave routing applications, the kinematic and diffusion waves should do the job in an accurate and forthright manner.

Notably, since a diffusion wave will actually calculate diffusion, including the case of zero diffusion, it follows that the solution of a diffusion wave encompasses the solution of a kinematic wave. Therefore, the diffusion wave is postulated as the flood wave *par excellence*, i.e., the type of wave generally indicated for use in practical applications of flood routing, analysis, and design.

11. CLOSING REMARKS

In general, mixed kinematic-dynamic waves, herein simply referred to as *mixed waves*, and which elsewhere have been widely referred to, albeit inaccurately, as "dynamic waves," are in fact not large enough nor permanent enough to veritably constitute flood waves. An accumulated body of theory and experience confirms this fact. On the other hand, kinematic waves and their close cousins, diffusion waves, typically feature large mass and are characteristically nondiffusive, i.e., either they are not attenuating or, else, attenuating only a very small amount; therefore, they are apt to be ideal models of flood waves. Since a numerical solution of a diffusion wave generally comprises that of a kinematic wave, the diffusion wave may be regarded as the most appropriate way to model flood waves.

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